Maximum Likelihood Identification of Aircraft Stability and Control Derivatives

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This paper presents the application of a generalized identification method for Flight Test Data Analysis. The Method is based on the Maximum Likelihood (ML) criterion and includes output error and equation error methods as special cases. Both the linear and nonlinear models with and without process noise are considered. The flight test data from lateral maneuvers of HL-10 and M2/F3 lifting bodies is processed to determine the lateral stability and control derivatives, instrumentation accuracies and biases. A comparison is made between the results of the output error method and the ML method for M2/F3 data containing gusts. It is shown that better fits to time histories are obtained by using the ML method. The nonlinear model considered corresponds to the longitudinal equations of the X-22 VTOL aircraft. The data is obtained from a computer simulation and contains both process and measurement noise. The applicability of the ML method to nonlinear models with both process and measurement noise is demonstrated. The effect of different control inputs on parameter accuracies is shown.

1. Introduction

THE importance of determining the aircraft stability and control derivatives from flight test data has been recognized for a long time. The other sources of these derivatives are wind-tunnel tests and, to a lesser degree, theoretical calculations. The extraction of stability and control derivatives from flight test data is a problem in parameter identification. An extensive literature exists on the parameter identification problem as is evidenced by a number of survey papers on the subject during the last five years.[2-4] The development of methods for flight test data analysis has proceeded somewhat separately from the development of similar techniques in other areas. A discussion of the principal methods currently in use for identifying stability and control derivatives from flight test data is given by Taylor, et al.[5] It is convenient to divide these methods into two categories: 1) Least Squares or Equation Error Methods, and 2) the Output Error Methods.

The Equation Error Methods (6) minimize the squared error in satisfying the equations of motion with respect to the unknown parameters. The main advantage of the Equation Error Methods is their computational simplicity. They are linear methods and generally do not require iterating on the data. The main disadvantage of Equation Error Methods is that they lead to biased estimates in the presence of measurement noise. For this reason, they are mostly used as start-up methods.

The Output Error Methods (5) minimize the error between the actual output (time histories) and the model output produced by using the same inputs. The optimization problem involved is a nonlinear one and requires use of iterative methods. The Output Error Methods have been applied successfully to flight test data not containing gusts or other flight disturbances. These methods, however, require specification of a weighting matrix and do not consider process noise in the performance index. The resulting estimates have large variances and may fail to converge.

The development of the Maximum Likelihood (ML) Method for Aircraft Parameter Identification has been motivated by several considerations. First, the Maximum Likelihood estimates have been shown to be asymptotically unbiased, consistent and efficient under mild regularity conditions.[4] Second, ML methods are more general and can handle both measurement and process noise. In cases where no process noise is present, and the covariance of the measurement noise is known, ML methods reduce to the Output Error Method. Similarly, in cases where no measurement noise is present, the ML Method reduces to the Equation Error Method if all the states are measured. Third, ML Methods have been found to yield realistic values of the variances of the parameters. Fourth, ML Methods can also be used to estimate the covariances of the noises. This eliminates the problem of specifying the weighting matrix or the covariance of the measurement errors, as in Output Error method or nonlinear filtering techniques for combined state and parameter estimation.

The arrangement of this paper is as follows. Section 2 discusses the ML Method. Sections 3 and 4 present results of applying ML Method to flight test data on HL-10 and M2/F3 lifting bodies. Section 5 includes results from a simulation of X-22 VTOL aircraft.

2. Maximum Likelihood (ML) Identification

The concept of ML identification using a Kalman filter was presented in Ref. 1. We discuss here certain extensions and numerical algorithms developed for aircraft parameter identification.

2.1 Linear Systems

Consider a multi-input multi-output linear system in state variable form:

$$\dot{x}(t) = Fx(t) + Gu(t) + \Gamma w(t) \tag{1}$$

$$y(t) = Hx(t) + Du(t) + v(t) \tag{2}$$

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Index category: Aircraft Handling, Stability and Control.

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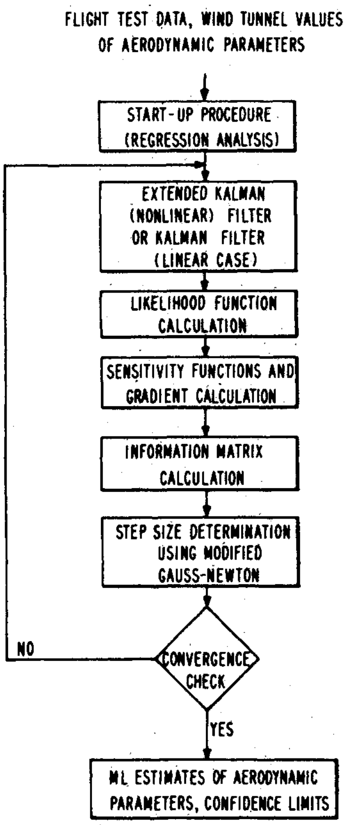

Fig. 1 Flow chart of the maximum likelihood indentication program.

where $x(t)$ is $n \times 1$ state vector, $u(t)$ is $p \times 1$ input vector, $w(t)$ is $q \times 1$ process noise vector (assumed zero mean and white with spectral density matrix Q),$y(t)$ is $r \times 1$ output vector sampled at discrete times $t = 1,2, \ldots, N$ and $v(t)$ is a $r \times 1$ measurement noise vector (assumed zero mean and white with covariance matrix R). Let Θ denote the $m \times 1$ vector of unknown but constant parameters in matrices F, G, Γ, D, Q and R and initial conditions $x(0)$. The negative log-likelihood function, $J(\Theta)$ for the above system is

$$J(\Theta) = \frac{1}{2}\sum_{t=1}^{N} \{\nu^T(t,\Theta)B^{-1}(t,\Theta)\nu(t,\Theta) + \ln |B(t,\Theta)|\} \quad (3)$$

and the ML estimate $\hat{\Theta}$ minimizes $J(\Theta)$ globally. In Eq. (3), $\nu(t,\Theta)$ and $B(t,\Theta)$ denote respectively the innovation sequence and its covariance as obtained from a Kalman filter for the given system. The Fisher information matrix, M is defined equivalently as $E[\partial^2 J/\partial\Theta^2]$ or $E[(\partial J/\partial\Theta)^T(\partial J/\partial\Theta)]$, both evaluated at $\hat{\Theta}$, where $E(\cdot)$ denotes expection over the sample space of N observations. After some tedious computations, it can be shown that the element in row j and column k of M is

$$M_{jk} = \sum_{t=1}^{N} \{E[(\frac{\partial\nu(t)^T}{\partial\theta_j}B(t)^{-1}\frac{\partial\nu(t)}{\partial\theta_k}] + \frac{1}{2}Tr[B(t)^{-1}\frac{\partial B(t)}{\partial\theta_j}B(t)^{-1}\frac{\partial B(t)}{\partial\theta_k}] \quad (4)$$

Notice that M^{-1} is the Cramer-Rao lower bound for the covariance of $\hat{\Theta}$.

2.2 Numerical Algorithm

We discuss here only a Modified Gauss-Newton method since a numerical comparison of different methods by one of the authors[7] and Bard[9] for minimizing $J(\Theta)$ has shown that the best convergence rate is achieved by this method.

The Modified Gauss-Newton method consists in updating parameters from iteration i to $i + 1$ as

$$\theta^{i+1} = \theta^i - \rho_i A_i \frac{\partial J}{\partial\theta}\bigg|_{\theta=\theta^i}$$

where ρ_i is a scalar step size parameter chosen to ensure that $J(\Theta^{i+1}) < J(\Theta^i) - \epsilon$, $\epsilon > 0$ and A_i is a positive definite matrix that approximates M^{-1}. This approximation consists in 1) dropping the expectation in Eq. (4), (2) evaluating terms in Eq. (4) at Θ_i and (3) for nearly singular M, computing the inverse, either by dropping eigenvalues of M below a positive small threshold (e.g. 10^{-6}) or shifting them to larger positive values. The details of computing A_i via singular value decompositions are discussed in Ref. 9, 10. A flow-chart of the ML algorithm is shown in Fig. 1.

2.3 Nonlinear Systems

The above numerical algorithm is general and can be used for nonlinear systems, as well. The exact calculation of the likelihood function, however, requires an optimal nonlinear filter. One can approximate such a filter by an Extended Kalman Filter[8] and assume the discrete innovations to be gaussian and white. Kailath[11] has shown that the distribution of the innovations $\nu(t)$ tends towards gaussian as the sampling rate is increased. Thus the above approximation is quite good for high sampling rates. This fact is also supported by simulation studies on the estimation of parameters in X-22 VTOL aircraft (see Section 5).

3. Linear Models Without Process Noise—HL-10 Lifting Body—Lateral Dynamics

An example of a straightforward application of the above method is the identification of the stability and control derivatives of a linearized dynamical model with no gusts. The equations of motion of the HL-10 lifting body provide such an example.

In this case, the maximum likelihood technique reduces to a form of the output error method, but with a specialized weighting matrix.

The linearized lateral equations of motion for the HL-10 are:

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p + \sin\alpha & Y_r - \cos\alpha & Y_\beta & \frac{g\cos\theta}{V} \\ 1 & \tan\theta & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} + \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix}$$

where α is the angle of attack, Θ is pitch angle, V is velocity, p is a roll rate, r is yaw rate, β is sideslip angle, ϕ is roll angle, δ_a is aileron deflection, δ_r is rudder deflection and C is a transformation from body to principal axis coordinate systems. The observations consist of all four states (p,r,β,ϕ) and the lateral acceleration

$$a_y = Y_\beta \cdot \beta + Y_{\delta a} \cdot \delta_a + Y_{\delta r} \cdot \delta_r + Y_o$$

Each of the observations are assumed to contain additive white gaussian noise of unknown covariance.

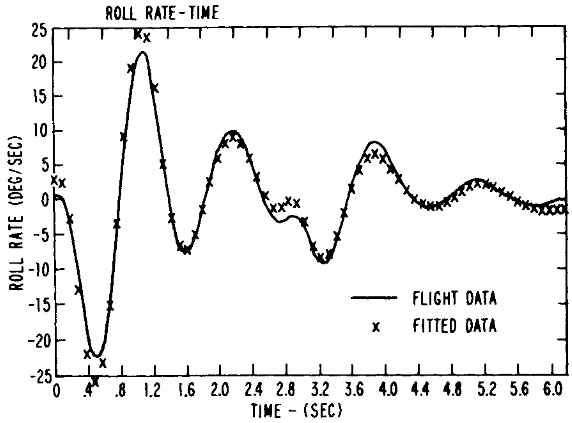

Fig. 2 Maximum likelihood identification of HL-10.

For the data record processed with the maximum likelihood identification technique, α, Θ, and V were kept constant. The total number of parameters, including the unknown initial conditions, is 22. The wind-tunnel values were used as the initial estimates for the parameters and the first data point was used as the starting estimate of the initial conditions. The measurement noise covariance matrix is estimated as

$$\hat{R} = [\frac{1}{N} \sum_{i=1}^{N} \nu_i \nu_i^T]$$

where N is the number of data points and ν_i is the vector of innovations or output errors.

The time histories shown in Figs. 2 and 3 were obtained after eleven iterations through the identification program. The fit in each of the observations is very good. A measure of the fit error or the performance index is $\ln|\hat{R}|$ where $|\cdot|$ denotes the determinant. Some of the parameter estimates, however, had the opposite signs from the wind tunnel values, and therefore did not appear to be reasonable. This situation of accurate fits but inaccurate parameter values occurs quite often in identification problems. There are several possible explanations for its occurrence in the aircraft example: 1) inadequate model (e.g. coupling between the lateral and longitudinal models is not being modeled, nonlinearities are neglected, etc.); and 2) inadequate excitation of all system models or lack of information in the data to identify all the parameters.

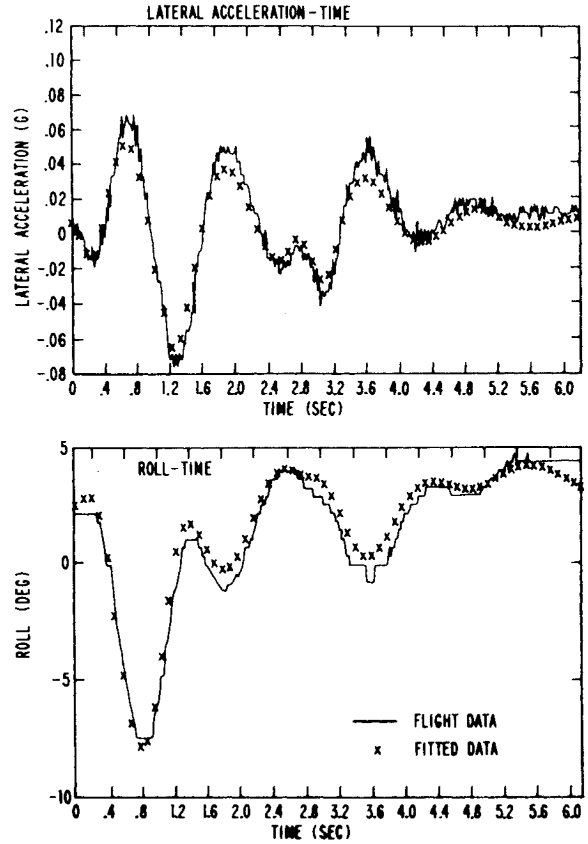

Fig. 3 Maximum likelihood identification of HL-10.

A method which was used to compensate for the incorrect signs was to add to the likelihood function a quadratic term involving the weighted differences between the es-

Table 1 HL-10 parameter estimates and standard deviations

Parameter	Wind tunnel and theoretical value	Max. lik. estimates (with st'd. dev.)	Max. lik. estimates with a priori weighting and bias (with st'd. dev.)
L_p	−0.3435	0.915 (0.025)	−0.271 (0.0114)
L_r	0.2723	−1.363 (0.138)	1.349 (0.0747)
L_β	−30.75	−56.489 (0.305)	−50.32 (0.330)
N_p	0.0245	−0.160 (0.00429)	0.0550 (0.00323)
N_r	−0.1290	0.432 (0.0187)	−0.0896 (0.0108)
N_β	6.8411	8.523 (0.719)	6.845 (0.0570)
Y_p	−0.0617	—	0.310 (0.00192)
Y_r	−0.0120	—	−1.019 (0.00762)
Y_β	−0.0916	−1.471 (0.0202)	−0.0918 (0.00382)
L_{δ_a}	11.2464	12.415 (0.0731)	12.282 (0.631)
L_{δ_a}	5.665	6.288 (0.173)	6.429 (0.114)
L_0	—	21.03 (0.302)	−0.0137 (0.0951)
N_{δ_a}	0.8135	1.262 (0.136)	1.357 (0.123)
N_{δ_r}	−3.617	−3.186 (0.033)	−3.194 (0.0212)
N_0	—	−1.633 (0.0479)	1.562 (0.0163)
Y_{δ_o}	−0.00180	0.0623 (0.00390)	−0.0482 (0.00443)
Y_{δ_r}	0.0111	0.919 (0.00992)	0.519 (0.00768)
Y_0	—	0.513 (0.00898)	−0.639 (0.00829)
p_{initial}	—	1.76 (0.158)	2.348 (0.117)
r_{initial}	—	0.117 (0.322)	−0.599 (0.0201)
β_{initial}	—	0.398 (0.0188)	−0.219 (0.0111)
ϕ_{initial}	—	1.939 (0.0697)	−1.429 (0.0571)
Negative log likelihood function value		−2243	−2264

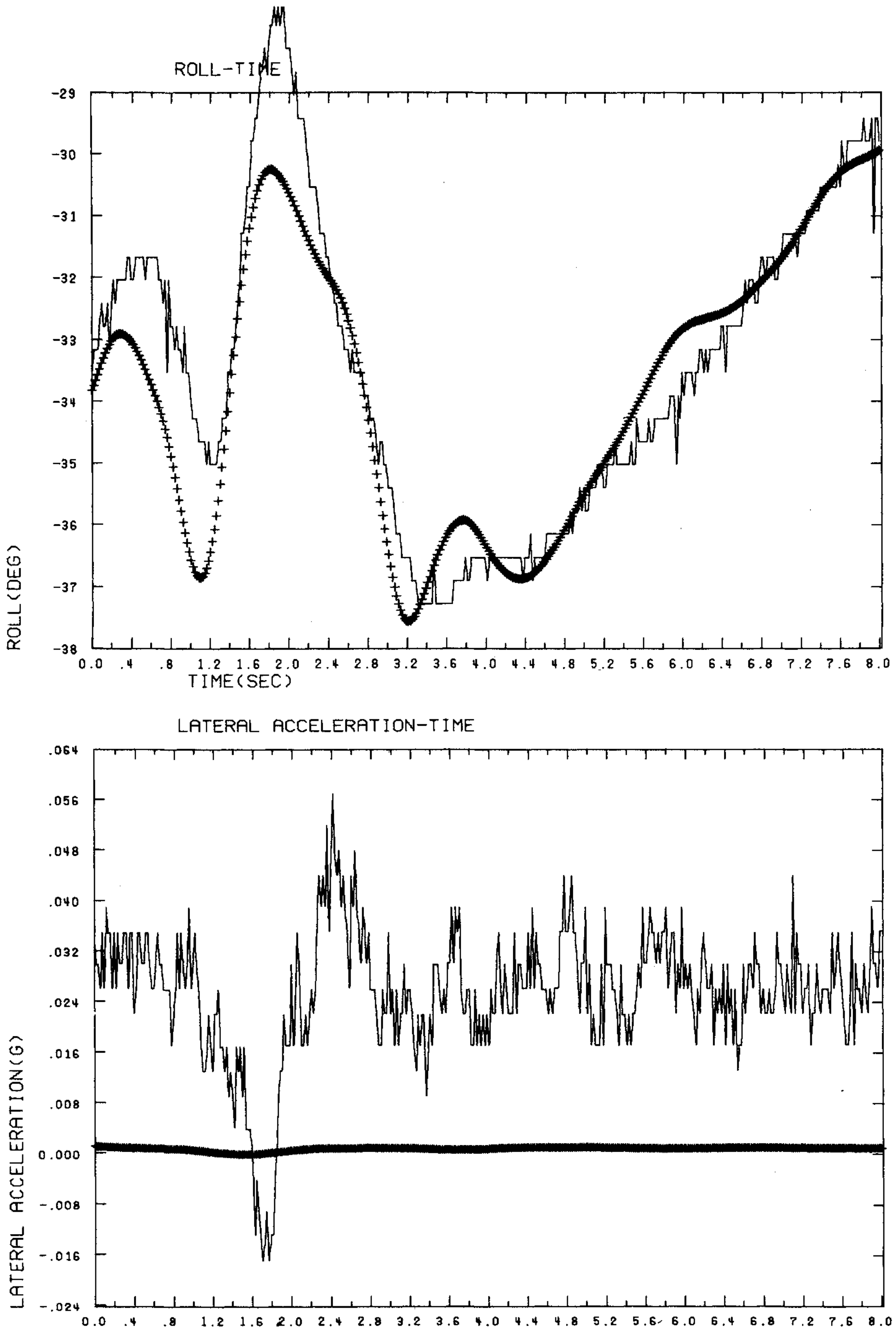

Fig. 4 Actual and fitted time histories using output error method for M2/F3.

timated and a priori parameter values. The fits to the observed data, using this a priori weighting, were biased. When the bias was identified and removed, the resulting fits were only slightly degraded from those using the ML techniques, in the output error mode. Several other corrections procedures are possible and they are described in the next section. A list of the parameter estimates is given in Table 1.

4. Linear Models With Process Noise: M2/F3 Lifting Body Lateral Dynamics With Gusts

A more interesting application of ML method is the identification of parameters in a linear model with process noise in the dynamical equations of motion.

The linearized equations of motion for the M2/F3 in the presence of gusts are

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p + \sin\alpha & Y_r - \cos\alpha & Y_\beta & \dfrac{g\cos\theta}{V} \\ 1 & \tan\theta & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} + \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_r} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix} + \begin{bmatrix} L_\beta \\ N_\beta \\ Y_\beta \\ 0 \end{bmatrix} \beta_n$$

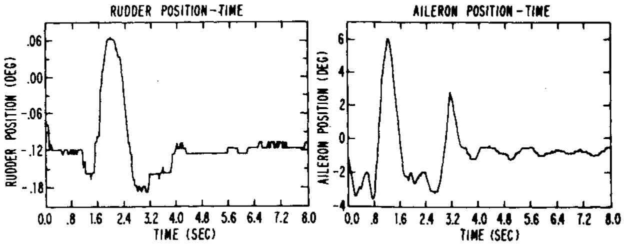

Fig. 5 M2/F3 control sequence time histories.

The β term is replaced by the $(\beta + \beta_n)$ term, where β_n represents the effect of gusts on the sideslip angle. The lateral acceleration is given by

$$a_y = Y_\beta(\beta + \beta_n) + Y_{\delta_a} \cdot \delta_a + Y_{\delta_r} \cdot \delta_r + Y_o$$

A priori, it is not known whether β_n is correlated or white noise.

To see the performance of the output error method in the presence of gusts, the M2/F3 data was first processed by the same method as was used for the HL-10. This assumed that $\beta_n = 0$. The time histories of two observations and their fits are given in Fig. 4. The fits are poor, especially in the lateral acceleration. The control inputs are shown in Fig. 6. Notice that the rudder input is very small and purely due to the Stability Augmentation System (SAS).

As a first step in applying the ML method, the measurement noise on the observation of the sideslip angle was assumed small compared to the gusts. The sideslip measurement y_β is therefore assumed equal to $(\beta + \beta_n)$ with no measurement noise.

This assumption leads to a particularly simple Kalman filter

$$\begin{bmatrix} \dot{\hat{p}} \\ \dot{\hat{r}} \\ \dot{\hat{\phi}} \end{bmatrix} = \begin{bmatrix} L_p & L_r & 0 \\ N_p & N_r & 0 \\ 1 & \tan\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{p} \\ \hat{r} \\ \hat{\phi} \end{bmatrix} + \begin{bmatrix} L_{\delta_a} & N_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix} + \begin{bmatrix} L_\beta \\ N_\beta \\ 0 \end{bmatrix} y_\beta$$

Where $\hat{p}$, $\hat{r}$, and $\hat{\phi}$ denote filtered estimates of p, r, and ϕ, and y_β is the measured sideslip angle.

The results of the maximum likelihood identification techniques with the effects of the gusts included in the above manner are given in Fig. 6. The fits in all the observations are extremely good, especially the fit to the lateral acceleration.

Using the parameter values obtained from this second processing of the M2/F3 data, an estimate of the time history of β can be derived (using the original β state equations). Subtracting this from the time history of y_β give an estimate of β_n. From this data, it was determined that a first order dynamical model for β_n was quite adequate and estimates for the time constant and process noise covariance were obtained from the autocorrelation function of β_n.

The estimates of the parameters are shown in Table 2, col. 2. It can be seen that some of the derivatives have changed sign from the wind-tunnel values. This is mainly attributed to the small excitation provided by the rudder deflection.

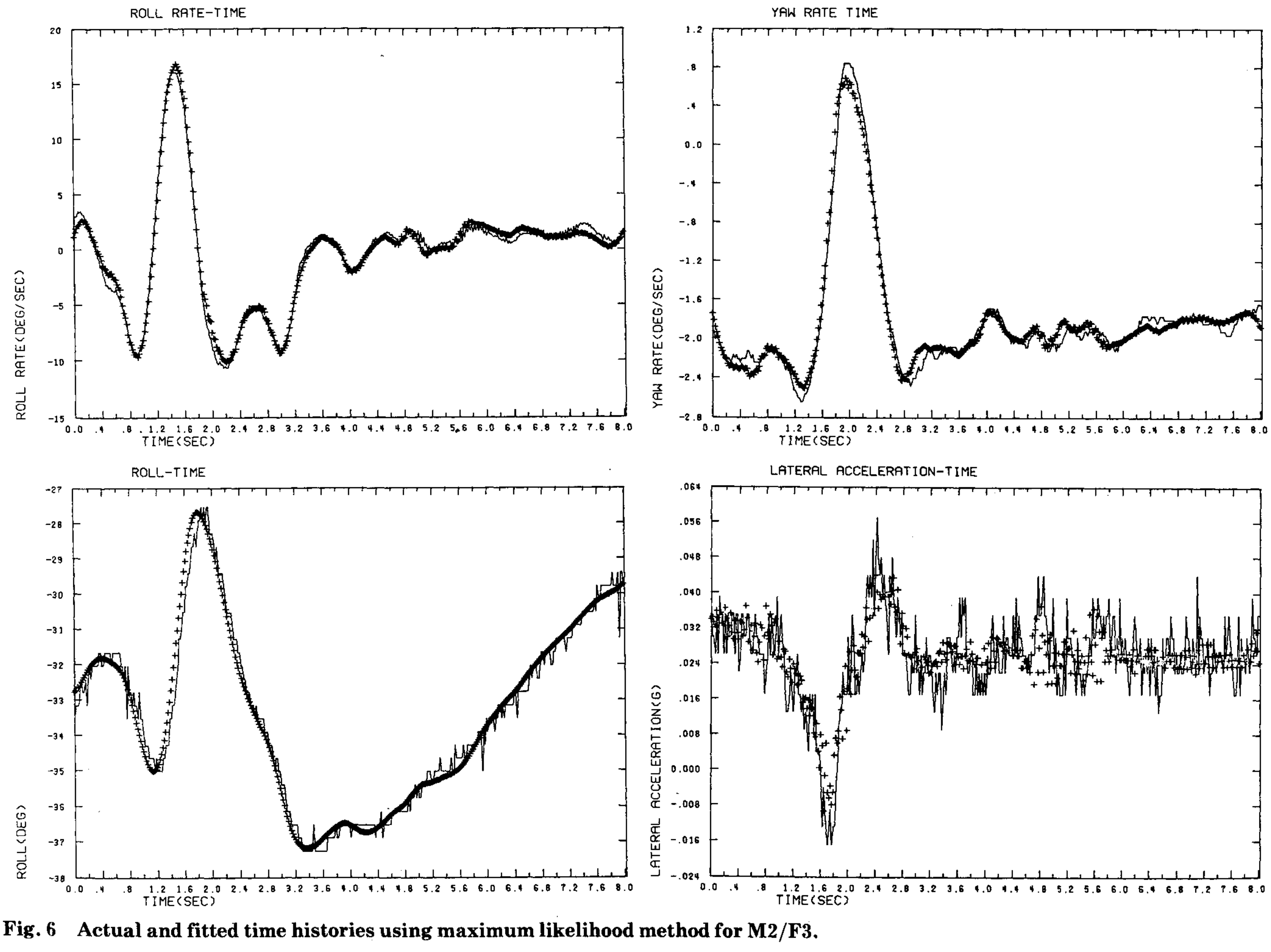

Fig. 6 Actual and fitted time histories using maximum likelihood method for M2/F3.

Table 2 M2/F3 parameter estimates and standard deviations

Parameter	Wind tunnel and theoretical	ML. estimate-output error mode (with st'd dev.)	ML. estimate assuming perf. β meas. (with st'd dev.)
L_p	−0.4673	−1.548 (0.0935)	0.679 (0.035)
L_r	0.8878	2.008 (1.187)	10.49 (0.547)
L_β	−75.140	−54.49 (2.45)	−97.79 (1.615)
N_p	0.0802	0.102 (0.006)	−0.0203 (0.00393)
N_r	−0.6876	−0.0307 (0.078)	−1.675 (0.0590)
N_β	7.5342	2.876 (0.136)	7.324 (0.152)
Y_p	*		
Y_r	*		
Y_β	−0.2001	−0.0476 (0.125)	−1.249 (0.0597)
L_{δ_a}	14.04	14.82 (0.301)	9.804 (0.109)
L_{δ_r}	10.03	73.97 (8.59)	−109.28 (5.519)
L_0	0	11.14 (1.828)	−10.46 (0.328)
N_{δ_a}	0.83	0.596 (0.0223)	0.719 (0.0104)
N_{δ_r}	−4.06	−12.874 (0.578)	6.844 (0.643)
N_0	0	−0.345 (0.121)	0.177 (0.03571)
Y_{δ_a}	0	−0.00033 (0.0151)	−0.0363 (0.00669)
Y_{δ_r}	0	0.0301 (0.363)	−0.874 (0.222)
Y_0	0	0.0179 (0.354)	0.378 (0.0299)
ϕ_{bias}			−0.281 (0.0531)
p_{initial}		3.807 (0.521)	0.359 (0.188)
r_{initial}		−2.262 (0.0785)	−1.66 (0.0280)
β_{initial}		−0.558 (0.0251)	*
ϕ_{initial}		−54.44 (0.175)	−32.69 (0.158)
Negative log likelihood function		−1502	−2237

Parameter	ML. estimates with a priori weighting (with st'd dev.)	ML. estimates with dependent params. fixed (with st'd dev.)	ML. with rank deficient solution
L_p	−0.461 (0.0182)	*	−0.531 (0.0189)
L_r	4.154 (0.140)	*	4.268 (0.144)
L_β	−67.95 (1.02)	*	−103.35 (0.105)
N_p	0.00475 (0.00349)	*	0.037 (0.00682)
N_r	−0.764 (0.0134)	*	−0.989 (0.0672)
N_β	6.763 (0.0876)	4.435 (0.113)	7.568 (0.306)
Y_p		*	*
Y_r		*	*
Y_β	−0.202 (0.00392)	−1.36 (0.0594)	−1.19 (0.0590)
L_{δ_a}	10.96 (0.161)	9.66 (0.169)	10.25 (0.0845)
L_{δ_r}	−42.18 (3.13)	*	−5.539 (0.0257)
L_0	−0.572 (0.115)	−9.004 (0.141)	−10.89 (0.280)
N_{δ_a}	0.762 (0.111)	0.756 (0.0134)	0.561 (0.0254)
N_{δ_r}	−4.37 (0.106)	*	−0.512 (0.651)
N_0	−0.233 (0.0433)	−0.00239 (0.0320)	0.587 (0.0833)
Y_{δ_a}	−0.0847 (0.00867)	−0.0275 (0.00634)	−0.0360 (0.00660)
Y_{δ_r}	−1.932 (0.286)	*	−0.737 (0.219)
Y_0	−0.0974 (0.378)	0.456 (0.0189)	0.408 (0.0296)
ϕ_{bias}	−6.01 (0.0933)	−0.667 (0.108)	−0.164 (0.0428)
p_{initial}	4.846 (0.296)	−3.125 (0.239)	−1.029 (0.139)
r_{initial}	−2.22 (0.0453)	−1.061 (0.0597)	−2.054 (0.0552)
β_{initial}	*	*	*
ϕ_{initial}	−31.91 (0.223)	−31.52 (0.224)	−31.576 (0.0705)
Negative log likelihood function	−1122	−1051	−1689

Three additional processing of the data were made using commonly applied techniques to improve the parameter estimates. The first was to constrain the optimization search so that those parameters that changed sign were constrained to have the correct sign. In most cases this only resulted in the parameter estimates coming out at the boundary values, usually 0.

The second attempt used the a priori weighting technique which has worked successfully for HL-10 data. However, for M2/F3 data the resulting fits to the observed data were poor although many of the derivative estimates (Table 2) were, in agreement with the wind-tunnel values. We did not carry out an extensive tuning of the a priori weighting parameters to improve these results.

The third approach was to fix, at an a priori value, all parameters for which output sensitivity was low or which showed strong correlation with other parameters in the information matrix. The results indicated that although the sign problem was removed (by fixing all those parameters causing problems), the fits to the observed data were unsatisfactory, as indicated by the value of the negative likelihood function in Table 2.

The solution to the parameter dependency problem which worked successfully was to find the directions in parameter space corresponding to combinations of parameters which could not be identified. A perfect dependency would, strictly speaking, result in a zero eigenvalue of the information matrix, causing it to be singular. However,

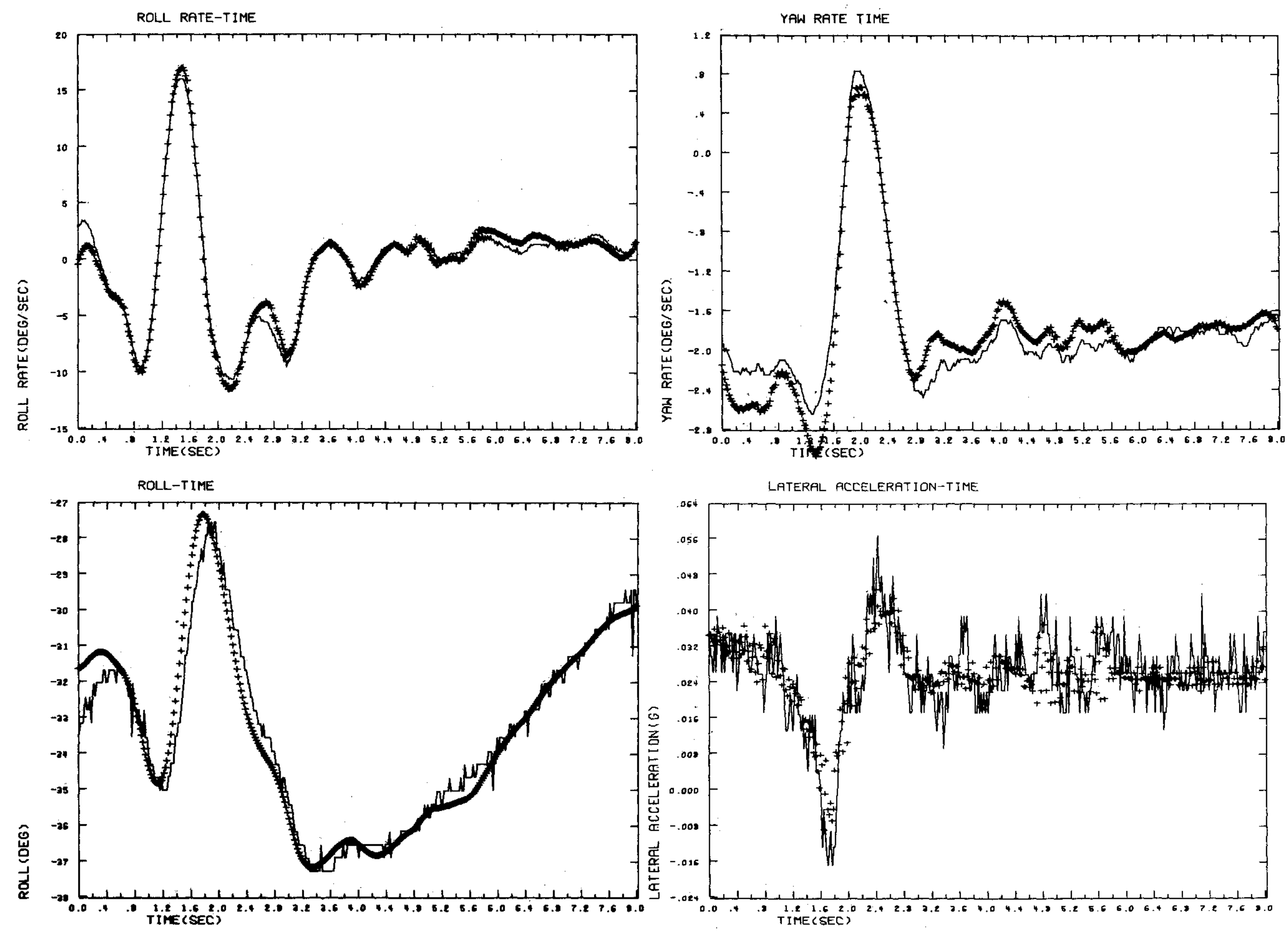

Fig. 7 Actual and fitted time histories using rank deficient maximum likelihood method for M2/F3.

since round-off and other numerical errors prevent the information matrix from being exactly singular, all the eigenvalues will be nonzero with a spread between the smallest and largest eigenvalue being many ordered of magnitude. In such a case, it is better to either use a rank deficient solution for the normal equations or shift the eigenvalue spectrum to make all eigenvalues positive and greater than a threshold value. Using the rank-deficient method, the fits to the observed data are shown in Fig. 7. Comparing these with Fig. 6, it can be seen that the fits are only slightly degraded. The parameter value obtained using third order rank deficient solution (3 eigenvalues deleted) are given in Table 2. Only $L_{\delta(n)}$ has sign opposite to wind-tunnel value and it is caused by iadequate excitation.

A second processing of the M2/F3 data was done to include the effect of measurement noise in y_β by adding a fifth state to represent β_n. The equations are given as

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0 & L_\beta \\ N_p & N_r & N_\beta & 0 & N_\beta \\ Y_p + \sin\alpha & Y_r - \cos\alpha & Y_\beta & q\cos\theta & Y_\beta \\ 1 & \tan\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} p \\ r \\ \beta \\ \phi \\ \beta_n \end{bmatrix} + \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega_\beta$$

where ω_β is white process noise in the β_n equation. The y_β measurement is written as $y_\beta = \beta + \beta_n + n_\beta$ where n_β is the measurement noise in y_β.

As outlined in Ref. 1, the maximum likelihood identification algorithm involves using the Kalman filter in the sensitivity equations rather than the original system equations. In this case, since there were five outputs and five states, the Kalman gain has twenty-five parameters; therefore, it was decided to identify the covariance of ω_β directly rather than identify the twenty-five Kalman gains.

The results substantiated the approximation that was made previously. The value for the variance of the measurement noise on sideslip angle came out close to zero and much smaller than the identified variance for the wind gust, β_n. This indicated that the original model for the data, with $y_\beta \cong \beta + \beta_n$ was, indeed, accurate.

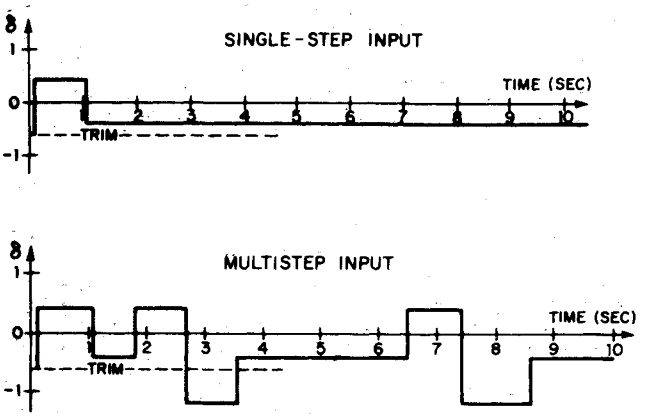

Fig. 8 Input time histories for X-22 identification.

5. Nonlinear Model with Process Noise—(X-22 VTOL —Longitudinal Dynamics)

The longitudinal dynamics of the X-22 VTOL aircraft is simulated as

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} x_w(u) - qw - g\sin\theta \\ z_w(u)w + qu + g\cos\theta \\ M_w(u)w + M_q(u)q \end{bmatrix} + \begin{bmatrix} x_o(u), x_{\delta_{es}}(u) \\ z_o(u), z_{\delta_{es}}(u) \\ M_o(u), M_{\delta_{es}}(u) \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ \delta_{es} \end{bmatrix} + \begin{bmatrix} x_u(u), x_w(u), 0 \\ z_u(u), z_w(u), 0 \\ M_u(u), M_w(u), M_q(u) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

where $x_0(u)$, $z_0(u)$, and $M_0(u)$ are second-order polynomials in u and $x_w(u), z_w(u)$ $M_w(u)$, $M_q(u)$, $x_{\delta es}(u)$, $z_{\delta es}$, $M_{\delta es}(u)$ are first-order polynomials in u. Furthermore, by definition $x_u(u) \, \Delta \, d/du(x_0(u) + x_w(u)w + x_{\delta es}(u)\delta_{es})$; $z_u(u)$ and $M_u(u)$ are similarly defined.

All the states and accelerations are measured so that

$$y^T(t) = [u(t), w(t), q(t), \theta(t), \dot{q}(t), n_x(t), n_y(t)] + v^T(t)$$

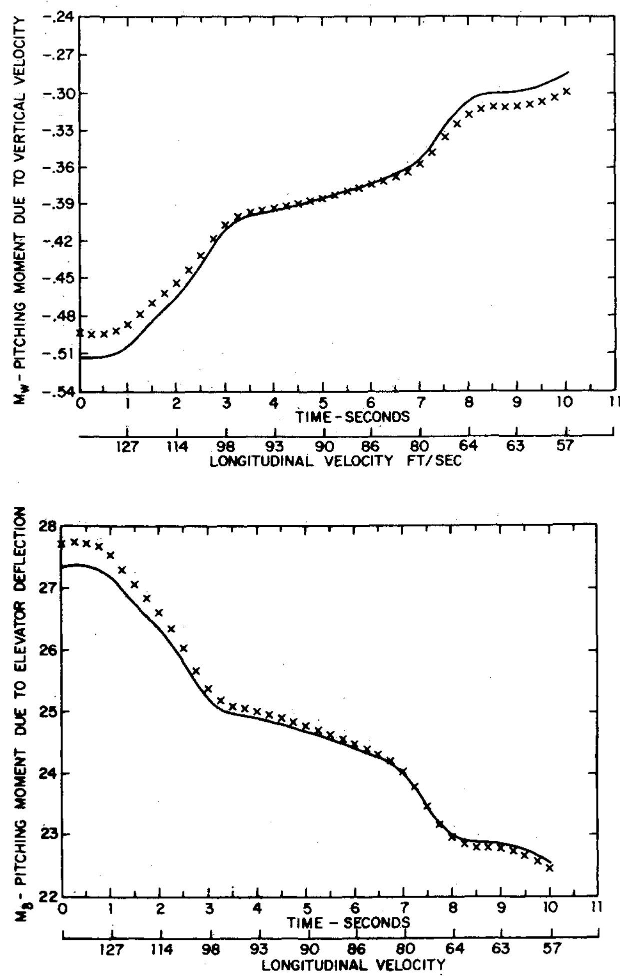

Fig. 9 Maximum likelihood identification of X-22 stability and control derivatives.

The sampling rate is 20/sec and the data length is 10 sec. The values of the process and measurement noise variances are $Q = \text{Diag}[1,1,.2](\Delta t)^2$, $\Delta t = 0.05$, $R = \text{Diag}[0.5,0.075,0.03,0.01,0.001,0.0005,0.0025]$. The input time-histories are shown in Fig. 8.

Table 3 Parameter estimates for X-22 VTOL model simulated data

Parameter	Correct parameter value	With single step input: Estimate	With single step input: St'd dev.	With multistep input: Estimate	With multistep input: St'd dev.
M_0	28.9	69.23	16.23	28.04	2.150
M_u	−0.176	−0.789	0.251	−0.159	0.0352
M_{u2}	−0.000355	0.00196	0.000984	−0.000441	0.000145
M_w	−0.100	−0.447	0.137	−0.0947	0.0251
M_{uw}	−0.00317	−0.000212	0.00119	−0.00321	0.000236
M_q	−0.497	−0.979	0.175	−0.489	0.0139
M_{uq}	−0.00103	−0.00267	0.00130	−0.00116	0.000152
M_δ	18.7	8.28	17.96	18.38	0.279
$M_{u\delta}$	0.0669	0.149	0.1392	0.0708	0.00306
X_0	18.3	22.66	4.771	18.88	0.873
X_u	−0.0917	−0.154	0.0669	−0.103	0.0132
X_{u2}	−0.0003	−0.0000756	0.000236	−0.00024	0.00005
X_w	0.221	0.177	0.0412	0.220	0.0105
X_{uw}	−0.00159	−0.00123	0.000341	−0.00159	0.00009
X_δ	−0.778	−1.087	7.072	−0.691	0.1139
$X_{u\delta}$	0.0184	0.0212	0.0547	0.0171	0.001134
Z_0	−32.2	−7.277	20.40	−35.15	2.631
Z_u	0.910	0.5678	0.2795	0.969	0.0427
Z_{u2}	−0.007	−0.00587	0.000968	−0.00728	0.000172
Z_w	−0.194	−0.566	0.164	−0.272	0.0323
Z_{uw}	−0.00287	−0.000699	0.00139	−0.00314	0.000321
Z_δ	−0.0351	−6.423	39.59	−0.673	0.388
$Z_{u\delta}$	0.0167	0.0681	0.3066	0.0200	0.00451

A number of cases were run and a comparison of the ML method with the Extended Kalman Filter for combined State and Parameter estimation was also made. The convergence of the ML algorithm, the parameter estimates and their standard deviations were much more satisfactory than those of the Extended Kalman Filter. Some of the results are shown in Figs. 9a and 9b and Table 3. It is seen that an order of magnitude improvement in standard deviations can be obtained by using the multistep input over the single-step input. The estimates of the total forces and moments (Fig. 9a and 9b) are also satisfactory, though the estimates of all the polynomial coefficients are not always as good. Further details on the results are contained in Refs. 7 and 8.

6. Conclusions

The generalized Maximum Likelihood Method which includes the Output Error Method and the Equation Error Method as special cases has been applied to flight test data from HL-10 and M2/F3 lifting bodies. Accurate fits to the time histories are obtained even in the presence of lateral gusts when the output error method leads to a very poor match with the time histories. The maximum likelihood method is extended to nonlinear dynamics with process noise and is applied to simulated data on X-22 longitudinal equations of motion. A marked improvement in identification from using a multistep input as opposed to a single step is demonstrated.

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The Cost of Noise Reduction in Intercity Commercial Helicopters

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The relationship between noise reduction and direct operating cost was studied for commerical helicopters having a design range of 400 miles. This was accomplished by generating a large number of helicopter preliminary designs with the aid of a computer program. Designs were selected to meet each of four noise level goals with minimum direct operating cost, establishing a curve of noise level vs direct operating cost. This was repeated for several payloads and technology time frames. It was concluded that good economic performance can be expected of relatively quiet future helicopters which have low tip speeds and high solidity rotors. With a 25% increase in direct operating costs the takeoff perceived noise level at 500 ft for a 1975, 50 passenger helicopter can be kept below 80 dB PNL. The expected improvements in helicopter technology over the next fifteen years can offset the economic penalties due to noise reduction.

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Index categories: VTOL Aircraft Design; VTOL Missions and Transportation Systems.

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Nomenclature

$\mu = (V)/(V_{\text{tip}})$ = rotor advance ratio
ρ_{cr} = air density in cruise regime, slugs/ft^3
ρ_h = air density in hover and low speed regime, slugs/ft^3
$\sigma = (A_B)/(R^2)$ = rotor solidity
ϕ = angle between the rotor axis, and a line joining the rotor hub, and a ground observer
A_B = total rotor blade area, ft^2
C = rotor blade mean chord, ft
$\bar{C}_L$ = rotor blade mean lift coefficient
C_T = rotor thrust coefficient = $(T)/(\rho\pi R^2 V_{\text{tip}}^2)$